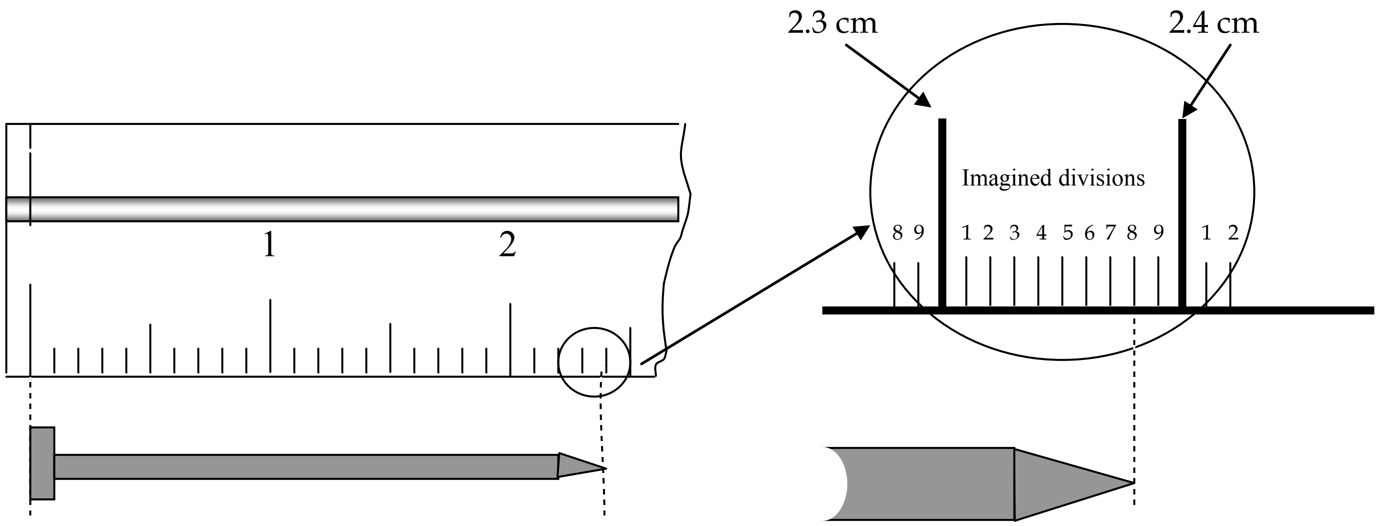
METRIC MEASUREMENT AND SIGNIFICANT FIGURES

UNCERTAINTY IN MEASUREMENT

The world is full of measurements and that holds true in science as well. People who do science are always interested in obtaining the most accurate data they can. A great deal of money is spent on instruments that can measure data to one more decimal place. So, whenever a measurement is made with a device such as a ruler or a graduated cylinder, an estimate is required in order to bleed out as much accuracy from the instrument possible yet still be accepted by the scientific community. This *estimation process* can be illustrated when measuring the pin shown below.



You can see from the ruler that the pin is a little longer than 2.3 cm and a little shorter than 2.4 cm. Because there are no graduations on the ruler between 2.3 and 2.4, you must estimate the pin's length between 2.3 and 2.4 cm. You do this by *imagining* that the distance between 2.3 and 2.4 is broken into 10 equal divisions and estimating to which of these imaginary divisions the end of the pin reaches. The end of the pin appears to be a little past half way between 2.3 and 2.4, which corresponds to 8 of your 10 imaginary divisions. So you estimate the pin's length as 2.38 cm. The result of your measurement is that the pin is *approximately* 2.38 cm in length, but you had to rely on a visual estimate, so it might actually be 2.37 or 2.39 cm.

Because the last number is based on a visual estimate, it may be different when another person makes the same measurement. For example, if five different people measured the pin, the results might be:

Person	Result of measurement
1	2.38 cm
2	2.36 cm
3	2.39 cm
4	2.38 cm
5	2.37 cm

Note that the first two digits in each measurement are the same regardless of who made the measurement; these are called the **CERTAIN** numbers of the measurement. However, the third digit is estimated and can vary; it is called an **UNCERTAIN** number. When you are making a measurement, the custom is to record all of the certain numbers plus the *first* uncertain number. It would not make any sense to try to measure the pin to the third decimal place (thousandths of a centimeter), because this ruler requires an estimate of even the second decimal place (hundredths of a centimeter).

It is very important to realize that *a measurement always has some degree of uncertainty*. The uncertainty of a measurement depends on the measuring device. For example, if the ruler, illustrated on the previous page, had marks indicating hundredths of a centimeter, the uncertainty in the measurement of the pin would occur in the thousandths place rather than in the hundredths place - but some uncertainty would still exist. At this point, do questions 1 – 5 on your activity sheet.

SIGNIFICANT FIGURES

The numbers recorded in a measurement (all of the certain numbers plus the first uncertain number) are called **significant figures**. The number of significant figures for a given measurement is determined by the inherent uncertainty of the measuring device. For example, the ruler used to measure the pin can give results only to the hundredths of a centimeter. Thus, when you record the significant figures for a measurement, you automatically give information about the uncertainty in the last number (the estimated number).

The uncertainty in the last number (estimated number) is usually assumed to be ± 1 unless otherwise indicated. For example, the measurement 1.86 kg can be interpreted as 1.86 ± 0.01 kg, where the symbol "±" means **plus or minus**. That is, it could be 1.86 kg – 0.01 kg = 1.85 kg or 1.86 kg + 0.01 kg = 1.87 kg.

Any measurement involves an estimate and thus is uncertain to some extent. You signify the degree of certainty for a particular measurement by the number of significant figures you record. Because doing science can require many types of calculations, You must consider what happens when you do arithmetic with numbers that contain uncertainties. It is important that you know the degree of uncertainty in the final result. Mathematicians have designated a set of rules to determine how many significant figures the result of a calculation should have. You should follow these rules whenever you carry out a calculation in this class. The first thing you need to do is learn how to count the significant figures in a given number. To do this, use the following rules designated by those mathematicians.

Rule #1: *Nonzero integers*. Nonzero integers (1, 2, 3, 4....etc.) *always* count as significant figures. For example, the number 1457 has four nonzero integers, all of which count as significant figures.

Rule #2: *Zeros* . There are three classes of zeros:

a. **Leading zeros** are zeros that *precede* all of the nonzero integers. They *never* count as significant figures. For example, in the number 0.0025, the three zeros simply indicate the position of the decimal point. The number has only two significant figures, the 2 and the 5.

b. **Captive zeros** are zeros that fall *between* nonzero integers. They *always* count as significant figures. For example, the number 1.008 has four significant figures (it has two captive zeros).

c. **Trailing zeros** are zeros at the right end of the number. They are significant only if the number is written with a decimal point. The number one hundred written a 100 has only one significant figure, but written as 100. –it has three significant figures.

Rule #3: *Exact numbers*. Often calculations involve numbers that were not obtained using measuring devices but were determined by counting: 10 experiments, 3 termites, 8 bean plants. Such numbers are called **exact numbers** - they can be assumed to have an *unlimited* number of significant figures. Exact numbers can also arise from definitions. For example,
1 inch is defined as exactly 2.54 centimeters. Thus, in the statement 1 in = 2.54 cm, neither 2.54 nor 1 limits the number of significant figures when it is used in a calculation.

Rules for counting significant figures also apply to numbers written in scientific notation. For example, the number 128 can be written as
1.28 x 102, and both versions have three significant figures. Scientific notation offers two major advantages: the number of significant figures can be indicated, easily, and fewer zeros are needed to write a very large or very small number. For example, the number 0.000060 is much more conveniently represented as 6.0×10^{-5}, and the number has two significant figures, written in either form.

ROUNDING OFF NUMBERS

When you perform a calculation on your calculator, the number of digits displayed is usually greater than the number of significant figures that the result should possess. So you must "round off" the number (reduce it to fewer digits)

Rules for rounding off:

Rule #1. If the digit to be removed is

a. less than 5, the preceding digit stays the same. For example 1.33 to two significant figures rounds to 1.3.

b. equal to or greater than 5, the preceding digit is increased by 1. For example, 1.36 to two significant figures rounds to 1.4, and 3.15 to two significant figures rounds to 3.2.

Rule #2. In a series of calculations, carry the extra digits through to the final result and *then* round off. This means that you should carry all of the digits that show on your calculator until you arrive at the final number (the answer) and then round off, using the procedures in rule 1.

One more point about rounding off to the correct number of significant figures. Suppose the number 7.348 needs to be rounded to two significant figures. In doing this, you look at only the first number to the right of the 3:

7.348

↑ look at this number (4) to round off to two significant figures.

The number is rounded to 7.3 because 4 is less than 5. It is incorrect to round sequentially. For example, do not round the 4 to 5 to give 7.35 and then round the 3 to 4 to give 7.4 as the final answer. When rounding off, *use only the first number to the right of the last significant figure* .

MULTIPLICATION AND DIVISION

For *multiplication* or *division* , the number of significant figures in the result is the same as that in the measurement with the *smallest number* of significant figures. The measurement with the smallest number of significant figures is the *limiting measurement* because it limits the number of significant figures in the result. For example, consider this calculation:

$$4.56 \times 1.4 = 6.384 \longrightarrow \text{Round off} \longrightarrow 6.4$$

Three significant figures | Two significant figures (limiting measurement | Two significant figures in the answer

Because 1.4 has only two significant figures, it limits the result to two significant figures. Thus the product is correctly written as 6.4, which has two significant figures. Consider another example. In the division

$$\frac{8.315}{298}$$

how many significant figures should appear in the answer?

Because 8.315 has four significant figures, the number 298 (with three significant figures) limits the result therefore, the answer should have only three significant figures. The calculation is correctly represented as

$$\frac{8.315}{298} = 0.0279027 \longrightarrow \text{Round off} \longrightarrow 0.0279 \quad \text{or} \quad 2.79 \times 10^{-2}$$

ADDITION AND SUBTRACTION

For *addition* or *subtraction*, the limiting term is the one with the smallest number of decimal places. For example, consider the following sum:

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   12.11
   18.0  ←—— Limiting term (one decimal place)
 + 1.013
 = 31.123 ——(Round off)——→ 31.1   (one decimal place)
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The correct result is 31.1 (it is limited to one decimal place because 18.0 has only one decimal place).

Consider another example:

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   0.6875 ←—— Limiting term (one decimal place)
 – 0.1
 = 0.5875 ——(Round off)——→ 0.6   (one decimal place)
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Complete questions 6 – 20 on your worksheet at this time

CONVERSIONS WITHIN THE METRIC SYSTEM

Measurement - To describe objects, scientists use units of measurement that are relative to the particular feature being studied. For example, centimeters or inches, instead of kilometers or miles, would be used to measure the dimensions of a sea shell; and kilometers or miles, rather than centimeters or inches, to measure the width of the Atlantic Ocean. Most areas of science have developed units of measurement that meet their particular needs. However, regardless of the unit used, all scientific measurements are defined within a broader system so that they may be understood and compared. In science, the fundamental units have been established by the International System of Units (SI; Table 1 below).

Unit	Quantity measured	Symbol
Meter	length	m
Kilogram	mass	kg
Second	time	s
Kelvin	temperature	K
Ampere	electric current	A
Mole	quantity of a substance	Mol
Candela	luminous intensity	cd

As stated in your previous science classes, one important advantage of the metric system is that it is based on "tens." As shown on the metric conversion diagram (Figure 1), conversion from one unit to another can be accomplished simply by moving the decimal point to the left if going to larger units or by moving the decimal point to the right if going to smaller units. For example, if you measure the length of a fish and it is 1.43 centimeters long, in order to convert its length to millimeters, start with 1.43 on the "centi-" step of Figure 4. Then move the decimal point one place (step) to the right (the "milli-" step). The length in millimeters becomes 14.3 millimeters.

Measurements and Conversions

Because the United States uses the English system and most of the rest of the world (and ALL scientists) use the metric system, we must be able to make measurements in the SI system. If you grew up in the United States, chances are pretty good that you have not mastered this skill! Complete question 21 on your worksheet at this time.

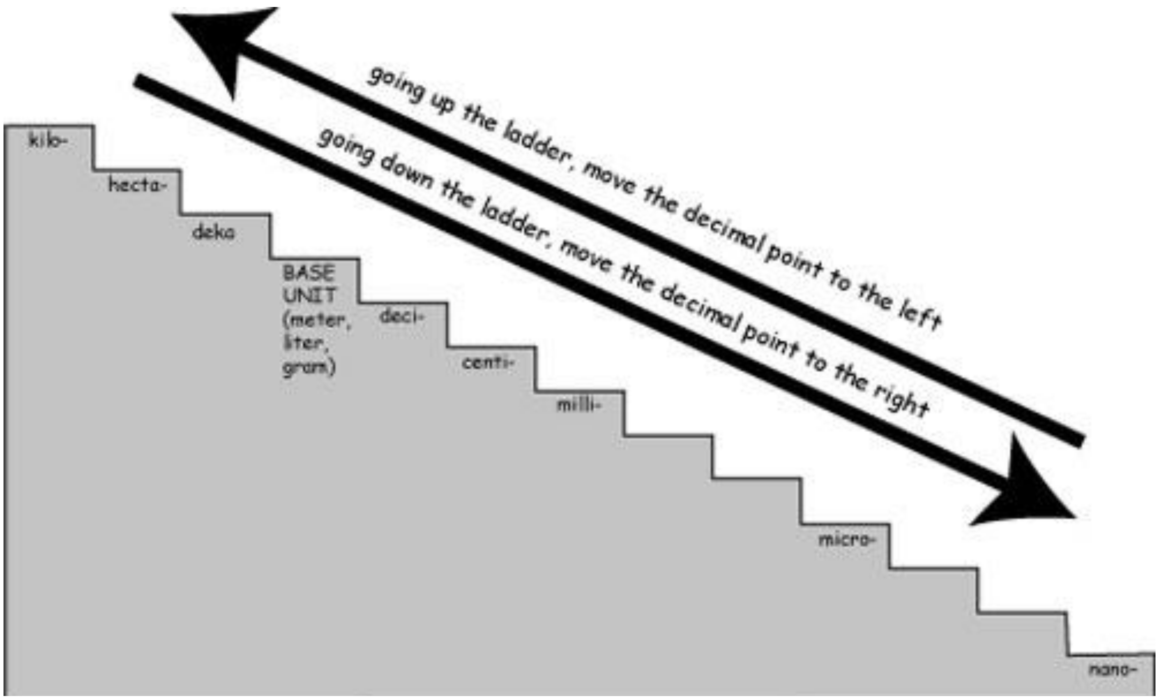


Figure 1. Metric conversion diagram. Beginning at the appropriate step, if going to larger units (left) move the decimal point to the left for each step crossed. When going to smaller units (right) move the decimal point to the right for each step crossed. For example, 1.253 meters (start at base unit step) would be equivalent to 1253.0 millimeters (decimal point moved three steps to the right to the “milli-“ step.

METRIC MEASUREMENT ACTIVITY

PRACTICE WITH UNCERTAIN NUMBERS

Using a class ruler and the concept of uncertain numbers, accurately measure each of the lines below and answer the questions for each line in the spaces provided. Make sure that you have the correct units in your measurement.

1. ____

a. What is the length of this line in centimeters (cm)?

b. From your answer, which of the numbers is/are *certain*?

c. Which of the numbers is/are *uncertain*?

2. ____

a. What is the length of this line in centimeters (cm)?

b. From your answer, which of the numbers is/are *certain*?

c. Which of the numbers is/are *uncertain*?

3. ______________________________

a. What is the length of this line in millimeters (mm)?

b. From your answer, which of the numbers is/are *certain*?

c. Which of the numbers is/are *uncertain*?

4. ___

a. What is the length of this line in centimeters (cm)?

b. From your answer, which of the numbers is/are *certain*?

c. Which of the numbers is/are *uncertain*?

5. Measure the width of this paper in centimeters _____________

Measure the length of this paper in centimeters _____________

Measure the diagonal of this paper in centimeters _____________

Measure the thickness of this paper in centimeters _____________

6. Indicate the number of significant figures in each of the following:

a. 1422 b. 65,321 c. 100,400

d. 200 e. 200. f. 20.0

g. 435.662 h. 56.341

7. Round off each of the following numbers to three significant digits.

a. 1,566,311 b. 0.0027651 c. 84,592

d. 0.0011672 e. 0.07759 f. 6.3049

8. Round each of the following numbers to the indicated number of significant digits.

a. 0.00034159 to three digits b. 10335.1 to four digits

c. 17.9915 to five digits d. 336,500 to three digits

9. Indicate the number of significant figures implied in each of the following statements:

a. The population of the United States is 250 million

b. One hour is equivalent to 60 minutes.

c. There are 5280 feet in 1 mile.

d. There are 1000 ml in one kiloliter.

e. The fish hatchery released 7,692 fingerling salmon into the river last year.

10. Using the concepts of significant figures and questions 1 – 5 above,

a. Subtract the length of the line in question #2 from the line length in question #1 __________

b. Subtract the length of the line in question #3 from the line length in question #1 ___________

c. Subtract the length of the line in question #4 from the line length in question #2 ___________

d. Add the length of the line in question #4 to the line length in question #2 ___________

e. Add the length of the line in question #3 to the line length in question #1 ___________

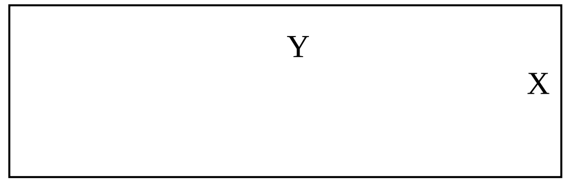
Use a ruler to measure each object below. Make sure that you utilize the uncertainty in measurement principles as well round all calculations to the correct significant figures. Include correct units of measure.

11. Given the shape (a) on the right,

a. What is the length of side X in cm? ________

b. What is the length of side Y in cm? _________

c. What is the area of the shape? _________

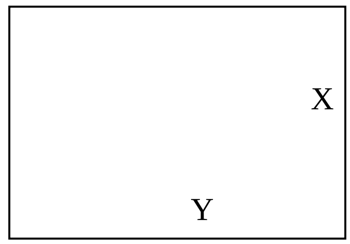
a. 

12. Given the shape (b) on the right,

a. What is the length of side X in cm? _________

b. What is the length of side Y in cm? _________

c. What is the area of the shape? _________

b. 

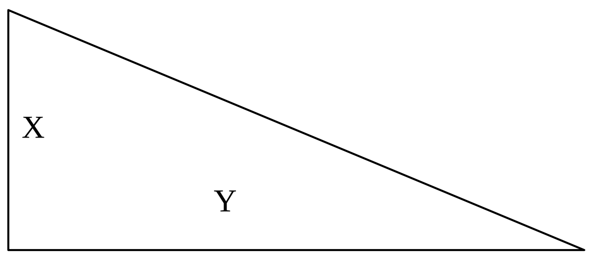
13. Given the shape on the right,

Equation: 1/2*bh* or 0.5XY

a. What is the length of side X in cm? ________

b. What is the length of side Y in cm? ________

c. What is the area of the shape? __________

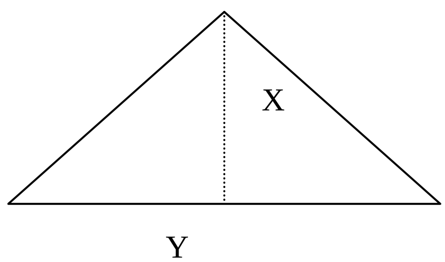


14. Given the shape on the right,

a. What is the length of side X in mm?

b. What is the length of side Y in mm?

c. What is the area of the shape?



15. When the calculation (0.0043)(0.821)(298) is performed, the answer should be reported to ___?___ significant figures.

16. The quotient (237.33)/(343) should be written with ___?___ significant figures.

17. How many digits after the decimal point should be reported when the calculation (199.0354 + 43.09 + 121.2) is performed?

18. How many digits after the decimal point should be reported when the calculation (10,434 – 9.3344) is performed?

19. Calculate each of the following and write the answer to the appropriate number of significant figures.

a. 212.2 + 26.7 + 402.9

b. 1.0028 + 0.221 + 0.10337

c. (52.331 + 26.01) – 0.9981

d. (201) + (3014)

20. Calculate each of the following and write the answer to the appropriate significant figures.

a. 0.08206 /0.995

b. (12.011)/(602.2)

c. (0.500)/44.02

d. 280.62/0.15

21. Make the following conversions using the step ladder technique demonstrated above.

a. 7.3 L to mL ________

b. 1875.7 mL to L ________

c. 1322.1 mL to L ________

d. 0.8 L to mL ________

e. 3048.8 L to mL ________

f. 4.6 L to mL ________

g. 0.8 mL to L ________

h. 9873.4 L to mL ________

i. 0.6 mL to L ________

j. 7.7 mL to L ________

k. 6.5 mL to L ________

l. 5551.5 L to mL ________

m. 7286.1 L to mL ________

n. 1.6 L to mL ________

o. 0.3 L to mL ________

p. 1.5 mL to L _____